

Exam 1 – 10/17/2022**Instructions**

- You have 50 minutes to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
3a	1	
3b	1	
4	1	
5	1	
Total		/ 100

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. You have been hired as an operations research analyst at Poisson Playland, a small amusement park just outside of Simplexville. According to your predecessor's notes, vehicles arrive at the entrance of the parking lot according to a stationary Poisson process at a rate of 17 per hour between 09:00 and 21:00.

a. What is the probability that 100 or fewer vehicles arrive at or before 15:00, given that exactly 75 vehicles arrive between 9:00 and 13:00?

b. What is the expected time of the 80th vehicle arrival?

Name:

c. What is the expected number of vehicles by the end of the day (09:00 - 21:00), given that exactly 50 vehicles arrive between 9:00 and 12:00?

d. Suppose it is 14:00. What is the probability that the next vehicle arrives within 5 minutes ($1/12$ hour)?

Problem 4. After the parking lot debacle, you start to suspect that your predecessor was wrong about the ticket booth as well. After collecting your own data, you find that customers (members and the general public together) actually arrive at the ticket booth according to a nonstationary Poisson process with the arrival rate function below:

$$\lambda(\tau) = \begin{cases} 32 & \text{if } 0 \leq \tau < 3 \\ 8 & \text{if } 3 \leq \tau < 9 \\ 12 & \text{if } 9 \leq \tau \leq 12 \end{cases}$$

where $\tau = 0$ corresponds to 9:00 and $\tau = 12$ corresponds to 21:00.

What is the integrated rate function for this nonstationary Poisson process?

Problem 5. The Simplexville Electric Company is conducting a study of its power line along the busiest part of Main Street. Looking at its historical data, the company has observed that power surges occur at a rate of 12 per hour, and this rate does not change over time. However, it also has noticed that the power surges come in “waves”: a large power surge is always followed by a smaller power surge exactly 1 minute later.

Professor I. M. Wright is consulting for the electric company. He thinks that the occurrence of power surges (both large and small) satisfies the independent increments property. Is he correct? Briefly explain.

Hint. It turns out that Professor I. M. Wright was your predecessor at Poisson Playland!