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SA402 · Dynamic and Stochastic Models

Exam 1 – 10/17/2022

Instructions	Problem	Weight	Score
• You have 50 minutes to complete this exam.	la	1	
• You may use your plebe-issue TI-36X Pro calculator.	1b	1	
• You may not use any other materials.	lc	1	
• No collaboration allowed All work must be your own	1d	1	
 Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow. 	2a	1	
	2b	1	
• Keep this booklet intact.	3a	1	
• Do not discuss the contents of this exam with any midshipmen until it is returned to you.	3b	1	
	4	1	
	5	1	
	Total		/ 100

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. You have been hired as an operations research analyst at Poisson Playland, a small amusement park just outside of Simplexville. According to your predecessor's notes, vehicles arrive at the entrance of the parking lot according to a stationary Poisson process at a rate of 17 per hour between 09:00 and 21:00.

a. What is the probability that 100 or fewer vehicles arrive at or before 15:00, given that exactly 75 vehicles arrive between 9:00 and 13:00?

b. What is the expected time of the 80th vehicle arrival?

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c. What is the expected number of vehicles by the end of the day (09:00 - 21:00), given that exactly 50 vehicles arrive between 9:00 and 12:00?

d. Suppose it is 14:00. What is the probability that the next vehicle arrives within 5 minutes (1/12 hour)?

Problem 2. The ticket booth at Poisson Playland is open from 09:00 to 21:00, and has two queues: one for members only, and one for the general public. Again, according to your predecessor, customers arrive at the members only queue at a rate of 7 per hour, and the general public queue at a rate of 23 per hour. Model the customer arrivals at both queues as independent, stationary Poisson processes.

a. What is the probability that the 49th customer (either member or general public) arrives at the ticket booth at or before 11:00?

b. What is the probability that 10 or more customers arrive at the members only queue during <u>each</u> of 2 consecutive hours? (For example, 10 or more customers arrive between 9:00 and 10:00, and then 10 or more customers arrive between 10:00 and 11:00.)

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Problem 3. After working at Poisson Playland for a few days, you start to suspect that your predecessor was wrong about the parking lot, so you collect your own data. You find that the vehicles actually arrive at the entrance of the parking lot according to a nonstationary Poisson process with the following integrated rate function:

$$\Lambda(\tau) = \begin{cases} 20\tau & \text{if } 0 \le \tau < 3\\ 5\tau + 45 & \text{if } 3 \le \tau < 9\\ 10\tau & \text{if } 9 \le \tau \le 12 \end{cases}$$

where τ = 0 corresponds to 9:00 and τ = 12 corresponds to 21:00.

a. What is the expected number of arrivals between 11:00 and 14:00?

b. What is the probability that 23 or more vehicles arrive between 17:00 and 19:00?

Problem 4. After the parking lot debacle, you start to suspect that your predecessor was wrong about the ticket booth as well. After collecting your own data, you find that customers (members and the general public together) actually arrive at the ticket booth according to a nonstationary Poisson process with the arrival rate function below:

$$\lambda(\tau) = \begin{cases} 32 & \text{if } 0 \le \tau < 3\\ 8 & \text{if } 3 \le \tau < 9\\ 12 & \text{if } 9 \le \tau \le 12 \end{cases}$$

where $\tau = 0$ corresponds to 9:00 and $\tau = 12$ corresponds to 21:00.

What is the integrated rate function for this nonstationary Poisson process?

Problem 5. The Simplexville Electric Company is conducting a study of its power line along the busiest part of Main Street. Looking at its historical data, the company has observed that power surges occur at a rate of 12 per hour, and this rate does not change over time. However, it also has noticed that the power surges come in "waves": a large power surge is always followed by a smaller power surge exactly 1 minute later.

Professor I. M. Wright is consulting for the electric company. He thinks that the occurrence of power surges (both large and small) satisfies the independent increments property. Is he correct? Briefly explain.

Hint. It turns out that Professor I. M. Wright was your predecessor at Poisson Playland!